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DISPERSION OF THERMAL WAVES IN GRANULAR MATERIAL

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The effective thermophysical parameters of a dispersed medium are discussed which characterize the propagation of temperature waves in the medium and the equations of nonstationary thermal conductivity are formulated.

Many papers (see [1-5], for example, and the review in [6]) have been devoted to a detailed study of nonstationary fields of temperature or impurity concentration in dispersed or other heterogeneous media. The interest in this subject is connected with the commercial prevalence of periodically operating equipment in which such media are used as working bodies and also of equipment in which there is a "response" to a sudden change in external conditions (chromatographic columns, absorbers, etc.). Nonstationary transport processes play an important role in phenomena occurring within the individual porous grains of a catalyst [7, 8] or in particles being dried [9], which can also be considered as a kind of heterogeneous material. Finally, such processes are important in laboratory practice in the determination of effective dispersion coefficients for heat or mass in composite materials and in dispersed flows of diverse structure [10,11].

Even for an analysis of the penetration of heat in the simplest "composite" material — a system of two adjacent uniform blocks [12] — or from the consideration of heat propagation along identically oriented fibers of an ordered fibrous material [13], it is clear that the behavior of a nonstationary temperature field in heterogeneous and homogeneous media differs not only in quantitative and qualitative respects, but also depends strongly on the structural features of the medium. The latter is responsible for the significant spread in the experimental data obtained under various conditions even in materials of identical structure together with the lack of a common viewpoint on the mechanism for transport processes in heterogenous media [5-10], with attempts at deriving some correlation relations that would generalize such data leading to extremely diverse results depending on the type of computational model used for the generalization [11]. Therefore, an a priori formal simulation of these processes is clearly unsatisfactory, and one feels the need for development of more detailed physical representations in the formulation of a deeper theory based on them.

We consider below only granular materials, one phase of which consists of discrete particles distributed in the other phase. In the general case, both phases of the medium may be mobile but the Péclet number

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characterizing local convective transport in the neighborhood of individual particles of the dispersed phase around which the continuous phase flow is assumed to be small so that only the convective transport associated with the mean motion of the medium is important.

The transport process in a dispersed medium of this kind is analyzed on the basis of the general methods developed in [14]. We further assume that the averaged characteristics of the medium vary in time and space considerably more slowly than the mean temperatures of its phases and that the linear scale L of the latter is considerably greater than the characteristic particle size a . These assumptions make it possible to use a continuous description of the transport process by considering the dispersed medium as some form of a spatially uniform continuum [14].

In addition, we assume for simplicity that the particles are identical spheres and that their volumetric concentration is small in comparison with unity. These assumptions do not affect the qualitative aspect of the effects discussed below and, in principle, can be eliminated, but they permit considerable simplification of the computations by focusing attention on the physical essentials of the matter. For definiteness, the subject of discussion is the propagation of heat, but all the results are equally valid for the propagation of a mass of impurity.

We first investigate a monochromatic temperature wave in which the time dependence of all quantities is concentrated in the factor $e^{i\omega t}$. The contact thermal conductivity over the body of particles in a dilute dispersed system can be neglected so that we have on the basis of [14] the equations

$$\begin{aligned} i\epsilon c_0 \omega \tau_0 &= -\nabla \mathbf{q} + \rho \Phi, & i\rho c_1 \omega \tau_1 &= -\rho \Phi, \\ \mathbf{q} &= -\lambda_0 \nabla \tau - (\lambda_1 - \lambda_0) \rho \mathbf{F}, & \tau &= \epsilon \tau_0 + \rho \tau_1 \end{aligned} \quad (1)$$

for the amplitudes τ_0 and τ_1 of the mean temperatures of the phases in a coordinate system coupled to the mean motion of the medium, where \mathbf{F} and Φ are the amplitudes of the linear functionals introduced in [14], which can be represented in the form

$$\mathbf{F} = \nu \nabla \tau_0, \quad \Phi = -\mu \tau_0 \quad (2)$$

with certain a priori unknown complex coefficients ν and μ .

In addition, \mathbf{F} and Φ can be expressed through quantities averaged over the dispersed phase in the form of the integrals [14]

$$\begin{aligned} \mathbf{F}(\mathbf{R}) &= \frac{3}{4\pi a^3} \int_{|\mathbf{R}-\mathbf{R}'| \leq a} \nabla \tau^*(\mathbf{R}, \mathbf{R}') d\mathbf{R}', \\ \Phi(\mathbf{R}) &= \frac{3}{4\pi a^3} \int_{|\mathbf{R}-\mathbf{R}'| \leq a} \nabla \mathbf{q}^*(\mathbf{R}, \mathbf{R}') d\mathbf{R}', \end{aligned} \quad (3)$$

where τ^* and \mathbf{q}^* are the amplitudes of the mean temperature and heat flow within a selected (test) particle and the integration is carried out over positions \mathbf{R}' of the center of that particle such that the point \mathbf{R} is within the particle.

We point out that Eqs. (3) are more exact than the analogous formulas obtained from Eqs. (3) by neglecting terms of higher order in a/L and discussed in detail in [14]. The equations mentioned express quantities averaged over the dispersed phase in the form of integrals over the surface, and not over the volume, of the test particle.

Equations (1) and (2) indicate that τ_1 is proportional to τ_0 , i. e., by introducing a proportionality factor σ , one can write

$$\begin{aligned} \tau_1 &= \sigma \tau_0, \quad \mathbf{q} = -\beta \lambda_0 \nabla \tau_0, \quad \mu = i\sigma c_1 \omega, \\ \beta &= \epsilon + \sigma \rho + (\kappa - 1) \nu \rho, \quad \kappa = \lambda_1 / \lambda_0, \end{aligned} \quad (4)$$

where the equation

$$i(\epsilon c_0 + \rho \sigma c_1) \omega \tau_0 = \beta \lambda_0 \Delta \tau_0 \quad (5)$$

holds for τ_0 .

Thus, the behavior of a monochromatic wave will be described completely if the coefficients β (or ν) and σ (or μ) are known. The latter must be found from a comparison of Eqs. (2) and (3) if one determines τ^* and q^* from a solution of the special problem involving the mean perturbations introduced by a test particle into the mean unperturbed fields τ_0 and τ_1 [14, 15].

In view of the smallness of the volumetric concentration of the particles, one can neglect their "lack of overlap," i.e., the fact that centers of neighboring spheres cannot be closer than a distance $2a$. This makes it possible to consider the local concentration of the dispersed phase at the surface of the test particle, which is discussed in [14], as simply corresponding to ρ and to simplify the calculations considerably. Further, because of the inequality $a \ll L$, it is appropriate to represent the field τ_0 in the neighborhood of the point \mathbf{R} in the form of a Taylor expansion, where it is sufficient to assume for the purposes of this paper that this field depends on some one Cartesian coordinate. Then

$$\tau_0(\mathbf{R} + \mathbf{r}) = \sum_{m=0}^{\infty} T_m(\mathbf{R}) z^m, \quad z = r \cos \theta. \quad (6)$$

Similar expansions can also be written in the neighborhood of all other points \mathbf{R}' , where the coefficients in such expansions (denoted below by T'_m) are simple polynomial functions of the coefficients T_m in Eq. (6) and of the components of the vector $\mathbf{R}' - \mathbf{R}$, the explicit form of which is not given here.

From Eqs. (5) and (6) we have the recurrence relation

$$T_m = \frac{i}{m(m-1)} \left(\frac{\xi}{a} \right)^2 T_{m-2}, \quad m \geq 2, \quad (7)$$

$$\xi = \zeta \eta, \quad \zeta^2 = \frac{\kappa}{\beta} \left(\varepsilon \frac{c_0}{c_1} + \rho \sigma \right), \quad \eta^2 = \frac{c_1 \omega}{\lambda_1} a^2,$$

where, as is easily seen, there occur the order-of-magnitude equalities

$$\xi \sim a/L, \quad aT_1 \sim \xi T_0. \quad (8)$$

In discussing the problem of a test particle with a center at the point \mathbf{R}' , it is convenient to use expansions in spherical functions also; the expansions

$$\tau_0(\mathbf{R}' + \mathbf{r}) = \sum_{m=0}^{\infty} M_m(\mathbf{R}', r) P_m(\cos \theta), \quad (9)$$

$$n \nabla \tau_0(\mathbf{R}' + \mathbf{r}) = \sum_{m=0}^{\infty} N_m(\mathbf{R}', r) P_m(\cos \theta)$$

are needed below, where M_m and N_m in Eqs. (9) are polynomials in r with coefficients that depend linearly on T'_m ; for brevity, these polynomials are not written out here.

The problem of the temperature field in the neighborhood of a test particle with a center at the point \mathbf{R}' ($r = 0$) has the form

$$\Delta \tau' - i(\xi/a)^2 \tau' = 0, \quad r > a; \quad \Delta \tau^* - i(\eta/a)^2 \tau^* = 0, \quad r < a, \quad (10)$$

$$\tau_0 + \tau' = \tau^*, \quad \beta n \nabla (\tau_0 + \tau') = \kappa n \nabla \tau^*, \quad r = a,$$

$$\tau' \rightarrow 0, \quad r \rightarrow \infty; \quad \tau^* < \infty, \quad r = 0,$$

where τ_0 is the unperturbed field defined by Eqs. (9). The solution of the problem (10) is written in the form

$$\tau' = \sum_{m=0}^{\infty} f_m(r) P_m(\cos \theta), \quad \tau^* = \sum_{m=0}^{\infty} \varphi_m(r) P_m(\cos \theta), \quad (11)$$

$$f_m(r) = A_m r^m \left(\frac{a}{\xi r} \right)^{m+1/2} H_{m+1/2} \left(\sqrt{-i} \frac{\xi r}{a} \right),$$

$$\varphi_m(r) = B_m r^m \left(\frac{a}{\eta r} \right)^{m+1/2} J_{m+1/2} \left(\sqrt{-i} \frac{\eta r}{a} \right),$$

with the coefficients A_m and B_m being expressed in the form

$$A_m = \frac{1}{\Delta_m} \left(\frac{\alpha}{\beta} M_m j_{m+1/2} - a N_m \frac{J_{m+1/2}}{\eta^{m+1/2}} \right),$$

$$B_m = \frac{1}{\Delta_m} \left(M_m h_{m+1/2} - a N_m \frac{H_{m+1/2}}{\xi^{m+1/2}} \right),$$

$$\Delta_m = a^m \left(\frac{J_{m+1/2}}{\eta^{m+1/2}} h_{m+1/2} - \frac{\alpha}{\beta} \cdot \frac{H_{m+1/2}}{\xi^{m+1/2}} j_{m+1/2} \right),$$

$$\begin{Bmatrix} h_{m+1/2}(x) \\ j_{m+1/2}(x) \end{Bmatrix} = \left(m + x \frac{d}{dx} \right) \frac{1}{x^{m+1/2}} \begin{Bmatrix} H_{m+1/2}(\sqrt{-i} x) \\ J_{m+1/2}(\sqrt{-i} x) \end{Bmatrix}.$$

Here $P_m(x)$, $J_m(x)$, and $H_m(x)$ are Legendre polynomials and Bessel and Hankel functions of the first kind; when $r = a$, the value of the argument is not written in the symbols for the various functions.

The above relations make it possible in principle to determine the quantities τ^* and q^* appearing in the integrals in Eqs. (3) to the order of the ratio a/L to as high a power as desired. We limit ourselves to terms no higher than fourth order in the final expressions for τ' and τ^* in Eqs. (11). As is easily seen, this means that in the expansions (6) and (9) for τ_0 and in the expansion for $n\nabla\tau_0$ in Eq. (9), it is necessary to keep only the first five or four terms, respectively. It is further clear from Eqs. (7) and (8) that the parameters ξ and η are of the order of a/L , i.e., one should assume $\xi \sim \eta \ll 1$. The last makes it possible to expand the Bessel and Hankel functions in Eqs. (11) and (12) in series in powers of their arguments, where it is only necessary to keep terms of order no higher than fourth in these series by limiting ourselves to the assumed accuracy.

As the result of rather lengthy computations, the temperature $\tau^*(\mathbf{R}|\mathbf{R}')$ is expressed in the form of a polynomial of the fourth degree in the parameters ξ and η , the coefficients of which are certain functions of the vector $\mathbf{r} = \mathbf{R} - \mathbf{R}'$ (in the present case of the quantities r and $\cos \theta = z/r$) and also depend linearly on the quantities M_m and N_m ($m = 0, \dots, 4$) introduced in Eqs. (9). The latter are represented in elementary fashion through r and the coefficients T'_m ($m = 0, \dots, 4$). Differentiation of the resultant polynomial makes it possible to express $\nabla\tau^*(\mathbf{R}|\mathbf{R}')$ and $\nabla q^*(\mathbf{R}|\mathbf{R}')$, which appear in the integrands in Eqs. (3), in the form of polynomials in ξ and η also.

In the latter polynomials, it is convenient to use in place of T'_m the coefficients T_m determined at the point \mathbf{R} under consideration in accordance with Eq. (6). Substituting in the specified polynomials the quantities T'_m expressed as functions of T_m and of the displacement vector $\mathbf{r} = \mathbf{R} - \mathbf{R}'$, using the recurrence relation (7) and the obvious relations

$$T_0(\mathbf{R}) \equiv \tau_0(\mathbf{R}), \quad T_1(\mathbf{R}) z_0 \equiv \nabla\tau_0(\mathbf{R}),$$

where z_0 is a unit vector in the direction of the z axis, we obtain from Eqs. (3)

$$\Phi(\mathbf{R}) = \frac{i\eta^2 \lambda_1}{a^2} \tau_0(\mathbf{R}) \left[1 + \frac{i\eta^2}{5} \left(3\xi^2 \frac{\alpha + \beta}{\alpha + 2\beta} - \frac{5\alpha + \beta}{3\beta} \right) \right],$$

$$\mathbf{F}(\mathbf{R}) = \frac{\nabla\tau_0(\mathbf{R})}{\alpha + 2\beta} \left[3\beta + \frac{i\eta^2}{15} \cdot \frac{S(\alpha, \beta)}{\alpha + 2\beta} + i^{3/2} \eta^3 \frac{\xi^3 \beta (\beta - \alpha)}{\alpha + 2\beta} \right],$$

$$S(\alpha, \beta) = \frac{\xi^2 \beta}{2\alpha + 3\beta} (44\alpha^2 + 59\alpha\beta - 13\beta^2) - (\alpha^2 + 13\alpha\beta + 4\beta^2)$$

(the intermediate computations and also the polynomial representations for τ^* and other quantities are not written down because of their extreme complexity). Equation (14) determines the intensity of the interphase heat transfer per unit volume of the medium and Eq. (15) gives the effective mean heat flow q from Eq. (1).

Comparing Eqs. (2) with Eqs. (14) and (15), we obtain equations for the determination of the unknown coefficients σ to β [the other coefficients introduced can be calculated from Eqs. (4)],

$$\sigma = 1 + \frac{i\eta^2}{5} \left(3\xi^2 \frac{\alpha + \beta}{\alpha + 2\beta} - \frac{5\alpha + \beta}{3\beta} \right),$$

$$\beta = \varepsilon + \sigma\rho + \frac{\rho(\alpha - 1)}{\alpha + 2\beta} \left[3\beta + \frac{i\eta^2}{15} \cdot \frac{S(\alpha, \beta)}{\alpha + 2\beta} + i^{3/2} \eta^3 \frac{\xi^3 \beta (\beta - \alpha)}{\alpha + 2\beta} \right].$$

It is reasonable to seek a solution of these equations in the form of polynomials in powers of η or, which is the same thing, in powers of $\sqrt{i\omega}$. To the accuracy assumed in this paper, one can set

$$\sigma = \sigma^{(0)} + i\omega\sigma^{(1)}, \quad \beta = \beta^{(0)} + i\omega\beta^{(1)} + (i\omega)^{3/2}\beta^{(2)}, \quad (17)$$

calculating the coefficients in these relations which are independent of the frequency ω by using the small parameter η .

In particular, we have from Eqs. (14)-(17)

$$\sigma^{(0)} = 1, \quad \beta^{(0)} = 1/4 \{2 - 3\rho - \kappa(1 - 3\rho) + [(2 - 3\rho - \kappa(1 - 3\rho))^2 + 8\kappa]^{1/2}\}. \quad (18)$$

The first of these relations demonstrates that under stationary conditions the mean temperatures of the phases agree with each other and with the mean temperature of the mixture as a whole, as should be expected. The second relation determines the effective stationary thermal conductivity of a granular material, which was previously calculated in [15].

Further, using Eq. (7), we have to the required accuracy

$$\begin{aligned} \sigma^{(1)} &= \frac{c_1 a^2}{5\lambda_1} \left(3\xi_0^2 \frac{\kappa + \beta^{(0)}}{\kappa + 2\beta^{(0)}} - \frac{5\kappa + \beta^{(0)}}{3\beta^{(0)}} \right), \quad \xi_0 = \xi|_{\sigma=1, \beta=\beta^{(0)}}, \\ \beta^{(1)} &= \frac{\rho}{K} \left[\sigma^{(1)} + \frac{c_1 a^2}{15\lambda_1} \cdot \frac{(\kappa - 1) S_0}{(\kappa + 2\beta^{(0)})^2} \right], \quad K = 1 - \frac{3\rho\kappa(\kappa - 1)}{(\kappa + 2\beta^{(0)})^2}, \\ \beta^{(2)} &= \frac{\rho}{K} \left(\frac{c_1 a^2 \xi_0^2}{\lambda_1} \right)^{3/2} \frac{(\kappa - 1)\beta^{(0)}(\beta^{(0)} - \kappa)}{(\kappa + 2\beta^{(0)})^2}, \quad S_0 = S(\kappa, \beta^{(0)}), \end{aligned} \quad (19)$$

where ξ , S , and $\beta^{(0)}$ are defined in Eqs. (7), (15), and (18), respectively. Equations (18) and (19) are considerably simplified in various limiting cases, particularly when $\kappa \rightarrow 0$ or $\kappa \rightarrow \infty$.

Note that when $\lambda_1 = \lambda_0$ ($\kappa = 1$) and $c_1 = c_0$, the particles are indistinguishable (with respect to heat transport) from the continuous phase. In this case, we obtain $\beta^{(0)} = 1$, $\xi_0 = 1$ from Eqs. (7) and (18) and $\sigma^{(1)} = \beta^{(1)} = \beta^{(2)} = 0$ from Eq. (19), which is typical of a homogeneous thermally conducting medium. In a heterogeneous granular medium, the effective "thermal conductivity" $\lambda = \beta\lambda_0$ and effective coefficient of interphase exchange display marked "non-Newtonian" properties in the general case; first, they depend on the wave frequency, and, second, they are complex quantities. This corresponds to two new effects which can significantly affect the behavior of thermal waves, influencing the frequency dependence of the rate of propagation of the waves, the phase-shift angles between the oscillations of the different variables, and, in the final analysis, the dispersion of wave packets and the response of temperature fields in the medium to changes in external temperature or heat flow. The nature of such an effect is easily explained by means of the simple equations (4) and (5) and therefore a study of it is omitted.

All the expressions given above can be considered as relating to Fourier transforms of arbitrary variables in the fields $\tau_0(t, r)$ and $\tau_1(t, r)$ by assuming

$$\tau_0(t, r) = \int e^{i\omega t} \tau_0(\omega, r) d\omega, \quad \tau_1(t, r) = \int e^{i\omega t} \tau_1(\omega, r) d\omega. \quad (20)$$

For these fields, we then obtain from the first equations in (4) and (16)

$$\tau_1(t, r) = \left(1 + \sigma^{(1)} \frac{\partial}{\partial t} \right) \tau_0(t, r). \quad (21)$$

Similarly, using Eq. (15) and considering q in Eq. (1) as a Fourier transform of the true mean heat flow $q(t, r)$, we obtain after simple transformation

$$q = -\lambda_0 \left(\beta^{(0)} + \beta^{(1)} \frac{\partial}{\partial t} \right) \nabla \tau_0 - \lambda_0 \frac{\beta^{(2)}}{\sqrt{\pi}} \int_{-\infty}^t \frac{\partial^2 \nabla \tau_0}{\partial t'^2} \frac{dt'}{\sqrt{t-t'}}. \quad (22)$$

It is easy to see that the quantities $|\beta^{(1)}/\beta^{(0)}|$ and $|\sigma^{(1)}|$ have the sense of certain characteristic relaxation times for thermal conductivity in the dispersed medium, the first of which characterizes the relaxation of the mean flow to a stationary value corresponding to a given instantaneous value of the gradient of the mean

temperature and the second of which characterizes the relaxation of the temperature of the dispersed phase to the temperature of the continuous phase as the result of heat exchange between the phases. From the physical point of view, a nonzero value for these times is obviously associated with a difference in the thermal inertia of the continuous and dispersed phases. Note that $\sigma^{(1)}$ from Eq. (19) can, in principle, be both positive and negative.

The last term for the mean heat flow in Eq. (22) depends on the history of the heat-transport process and is nonlocal in time. In this respect, it reminds one, for example, of the well-known Basset force acting on a small particle suspended in a pulsating flow. As far as the authors know, the existence of such a term has not been pointed out previously for media of the type discussed here.

As follows from Eqs. (1) and (4), $\tau_1(t, \mathbf{r})$, and also the mean temperature $\tau(t, \mathbf{r})$ of the medium as a whole, are expressed in the form of simple linear functionals of $\tau_0(t, \mathbf{r})$, i.e., it is only necessary to determine the field $\tau_0(t, \mathbf{r})$. An equation for the direct determination of this field should be obtained after the application of the inverse Fourier transform to Eq. (5). If we limit ourselves to terms of the order of ω [or, which is the same thing, to the order of $(a/L)^2$], we obtain from Eq. (5)

$$(\epsilon c_0 + \rho c_1) \frac{\partial \tau_0}{\partial t} = \beta^{(0)} \lambda_0 \Delta \tau_0, \quad (23)$$

i.e., in this approximation, the dispersed medium is indistinguishable from some homogeneous medium, the thermal conductivity and heat capacity of which are, respectively, $\beta^{(0)} \lambda_0$ and $\epsilon c_0 + \rho c_1$.

To an accuracy of the order of ω^2 [or $(a/L)^4$], we have from Eq. (5)

$$\rho \sigma^{(1)} c_1 \frac{\partial^2 \tau_0}{\partial t^2} + (\epsilon c_0 + \rho c_1) \frac{\partial \tau_0}{\partial t} = \lambda_0 \left(\beta^{(0)} + \beta^{(1)} \frac{\partial}{\partial t} \right) \Delta \tau_0, \quad (24)$$

from which the sense of the quantities $|\beta^{(1)}/\beta^{(0)}|$ and $|\sigma^{(1)}|$ as relaxation times becomes especially clear. Terms which depend on the history of the process and which thereby complicate analysis of heat transport considerably only appear to the approximation with respect to ω ; this should be considered as an extremely favorable circumstance.

It is significant that Eq. (24) can be either hyperbolic or elliptic depending on the sign of $\sigma^{(1)}$ if one neglects the relaxation of thermal flow by setting $\beta^{(1)} = 0$ (although there is no justification for doing so). Obviously, the first case is encountered considerably more often so that certain advances achieved in the description of heat transport in dispersed media by means of an equation of the hyperbolic type [16] become understandable. As far as the authors know, elliptic equations have not been included in the simulation of heat transfer previously. In the general case, of course, it is necessary to consider all terms appearing in Eq. (24).

We point out in conclusion that Eq. (24) and the parameters σ and β were obtained here under definite, and sometimes limited, assumptions. However, one can assume from general considerations that this equation is applicable to the analysis of transport processes in a considerably broader class of heterogeneous materials, although in this case it is necessary to consider $\beta^{(1)}$ and $\sigma^{(1)}$ as empirical coefficients. In particular, precisely such an equation should be used for correlation analysis of experimental data and in the construction of computational models.

NOTATION

A_m, B_m , coefficients in (11), (12); a , particle radius; c , heat capacity per unit volume; F , functional introduced in (2), (3); f_m, φ_m , functions in (11); h_m, j_m , functions defined in (12); L , scale of mean temperature fields; M_m, N_m , coefficients in (9); \mathbf{n} , unit vector, \mathbf{r}/r ; \mathbf{q} , heat flow; \mathbf{R}, \mathbf{R}' , \mathbf{r} , spatial coordinates; S , quantity in (15); T_m , coefficients in (6); t , time; z , coordinate on which mean temperature fields depend; β , parameter defined in (4); Δ_m , parameter in (12); ζ, η , parameters introduced in (7); ϵ , porosity; θ , polar angle; $\kappa = \lambda_1/\lambda_0$; λ , thermal conductivity; μ, ν , coefficients in (2); ξ , parameter in (7); ρ , volumetric concentration of particles; σ , proportionality factor introduced in (4); τ , mean temperature or its amplitude; Φ , functional defined in (2), (3); ω , frequency. Indices: 0, 1, quantities associated with the continuous and dispersed phases, respectively; asterisks, temperature and heat flow within a test particle.

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MECHANICS OF JET FLOWS IN GRANULAR LAYERS.
 COALESCENCE OF BUBBLES IN CONSTRAINED
 FLOW CONDITIONS

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The coalescence of bubbles forming during the injection of a system of parallel jets into a high fluidized layer is investigated. The effect of the parameters of perforated gas-distributor arrays on the formation of the layer structure is briefly discussed.

A system of parallel jets is very often used for fluidization of granular layers and also for improving the characteristics of layers fluidized by an independent homogeneous flow. In both cases the characteristics of these jets have a marked effect on the formation of the layer structure as well as on the intensity of the processes of heat and mass transfer realized in the layer. From the point of view of applications the main interest lies in the interaction of jets and the bubbles forming in them and in their dependence on the characteristics of the layer itself as well as on the initial parameters of the jets (shape and size of nozzles or apertures, velocity, the step between adjacent apertures, and so forth). It is just this interaction that primarily determines the nature of gas distribution and the required structure of the fluidized layer so that its investigation is entirely necessary for developing methods of layer structure control and engineering techniques of its computation, as well as for the construction of gas-distributor units.

In spite of the obvious practical significance of this problem, its meaningful investigation is still in a rudimentary stage (for example, see [1]). There are only isolated empirical or purely engineering investigations of particular problems encountered in the construction or operation of certain equipment. Theoretically, constrained motions in a fluidized layer have been investigated only in connection with the restricting effect of the equipment walls on the distribution of gas flows around a solitary bubble [2], with the interaction of two closely spaced bubbles in an infinite layer [3], and with the mutual effect of two stationary adjacent plane jets on gas injection and the particles in each of these jets [4]. Below, the results of experiments on the investigation of the interaction of parallel jets in a high layer and on the determination of the height of primary coalescence of bubbles as a function of the physical and regime parameters are presented and discussed.

A system of two semiinfinite vertical jets flowing out into a fluidized layer of particles of polystyrene, nitroammophosph (a nitrogen-ammonium-phosphorus fertilizer), aluminosilicate catalyzer, and sand of different granulometric composition was taken as the initial objects for investigation. In most experiments the

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